# A-LEVEL MATHEMATICS - PREPARATORY WORK 2023

The purpose of this transition work is to:

- Consolidate the GCSE content required at A-level
- Address any gaps from GCSE
- Develop independent study skills

Each section includes links to video lessons that cover the topics required, written examples and practice questions. I would recommend reading the examples and then using the videos for further explanation if required. You do NOT have to watch all the videos they are there for additional support/ guidance. If you are struggling with a particular area, you can find additional practice on the Corbett Maths website. **For each section you are required to complete the practice questions on file paper, with a clear title and all working shown.**

The answers to each unit can be found at the back of this booklet. Please ensure you mark your work and then complete the Transition Pack Review Sheet. You get 1 mark per correct answer.

You will be assessed on your understanding of the subject content covered in this transition pack at the start of term 1.

Please ensure you bring the completed transition work and review sheet with you on your first lesson.

## Transition Pack Review Sheet

| **Section** | | **Score** | **RAG** | **Comments** |
|---|---|---|---|---|
| Expanding brackets and simplifying expressions | 21 | | | |
| Surds and rationalising the denominator | 14 | | | |
| Rules of indices | 30 | | | |
| Factorising expressions | 20 | | | |
| Completing the square | 8 | | | |
| Solving quadratic equations by factorisation | 10 | | | |
| Solving quadratic equations by completing the square | 4 | | | |
| Solving quadratic equations by using the formula | 4 | | | |
| Solving linear simultaneous equations by elimination | 4 | | | |
| Solving linear simultaneous equations by substitution | 5 | | | |
| Solving linear and quadratic simultaneous equations | 4 | | | |
| Linear inequalities | 9 | | | |
| Algebraic Fractions | 6 | | | |
| Rearranging Formulae | 12 | | | |

Areas for development:

Teacher comments:

# A - LEVEL MATHEMATICS 2023

## STRUCTURE OF THE COURSE

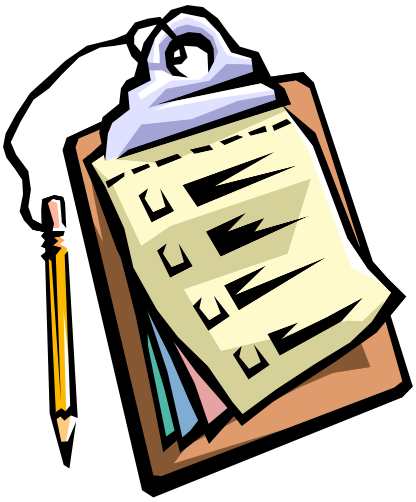
- Your A-Level Maths course covers Pure Mathematics, Mechanics and Statistics.
- You will be examined at the end of the two-year course. The assessment will consist of three two-hour exams.

## WORKLOAD & ORGANISATION

- You will have a heavy workload. Class time will be spent giving notes and explanations with questions and reinforcement set as homework.
- For <u>each</u> hour of taught time you should expect to spend an hour on maths work outside of lessons.
- You will be given exercise books for class notes, assessments and independent study. Class work will be completed on paper and you will need to keep this organised.
- Each week you will be set homework by both of your teachers. In addition, you will be expected to complete an independent exercise of your choice.

## HOW TO BE A SUCCESSFUL A LEVEL STUDENT

The most successful students at A-level Mathematics:



- Recognise that A Level Mathematics is demanding and that they will have to work hard to understand the course fully (even if up until now they have always found maths easy)
- Participate in lessons, answering questions and asking questions about the ideas studied
- Ensures notes are well organised and include the relevant detail and explanation and use their notes to help them do the work set
- Seek help outside lessons before the deadline when they get stuck (despite having used their notes and had a go)
- Submit work on time that is complete
- Revise thoroughly with a focus on avoiding previous misconceptions / mistakes.

# Expanding brackets and simplifying expressions

| **A LEVEL LINKS** | |
|---|---|
| Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds | |
| **Video Links** | |
| Corbett Maths | Hegarty Maths on Youtube |
| 13 Algebra: Expanding Brackets<br>14 Algebra: Expanding Two Brackets<br>15 Algebra: Expanding Three Brackets | |

## Key points

- When you expand one set of brackets you must multiply everything inside the bracket by what is outside.
- When you expand two linear expressions, each with two terms of the form $ax + b$, where $a \neq 0$ and $b \neq 0$, you create four terms. Two of these can usually be simplified by collecting like terms.

## Examples

**Example 1** Expand $4(3x - 2)$

| | |
|---|---|
| $4(3x - 2) = 12x - 8$ | Multiply everything inside the bracket by the 4 outside the bracket |

**Example 2** Expand and simplify $3(x + 5) - 4(2x + 3)$

| | |
|---|---|
| $3(x + 5) - 4(2x + 3)$<br>$= 3x + 15 - 8x - 12$ | **1** Expand each set of brackets separately by multiplying $(x + 5)$ by 3 and $(2x + 3)$ by $-4$ |
| $= 3 - 5x$ | **2** Simplify by collecting like terms: $3x - 8x = -5x$ and $15 - 12 = 3$ |

**Example 3** Expand and simplify $(x + 3)(x + 2)$

| | |
|---|---|
| $(x + 3)(x + 2)$<br>$= x(x + 2) + 3(x + 2)$<br>$= x^2 + 2x + 3x + 6$ | **1** Expand the brackets by multiplying $(x + 2)$ by $x$ and $(x + 2)$ by 3 |
| $= x^2 + 5x + 6$ | **2** Simplify by collecting like terms: $2x + 3x = 5x$ |

**Example 4** Expand and simplify $(x - 5)(2x + 3)$

| | |
|---|---|
| $(x - 5)(2x + 3)$<br>$= x(2x + 3) - 5(2x + 3)$<br>$= 2x^2 + 3x - 10x - 15$ | **1** Expand the brackets by multiplying $(2x + 3)$ by $x$ and $(2x + 3)$ by $-5$ |
| $= 2x^2 - 7x - 15$ | **2** Simplify by collecting like terms: $3x - 10x = -7x$ |

**Example 5** Expand and simplify $(x - 5)(2x + 3)(x - 3)$

| | |
|---|---|
| $(x - 5)(2x + 3)$<br>$= 2x^2 - 7x - 15$ | **1** Expand the first two brackets $(x-5)(2x + 3)$ |
| $(2x^2 - 7x - 15)(x - 3)$<br>$= 2x^2(x - 3) - 7x(x - 3) - 15(x - 3)$<br>$= 2x^3 - 6x^2 - 7x^2 + 21x - 15x + 45$<br>$= 2x^3 - 13x^2 + 6x + 45$ | **2** Multiply the answer by the final bracket, $(2x^2 - 7x - 15)(x - 3)$<br><br>**3** Simplify by collecting like terms |

# Practice

> **Watch out!**
> When multiplying (or dividing) positive and negative numbers, if the signs are the same the answer is '+'; if the signs are different the answer is '–'.

**1** Expand.

**a** $3(2x - 1)$

**b** $-(3xy - 2y^2)$

**2** Expand.

**a** $4k(5k^2 - 12)$ **b** $-2h(6h^2 + 11h - 5)$

**3** Expand and simplify.

**a** $3(y^2 - 8) - 4(y^2 - 5)$ **b** $4p(2p - 1) - 3p(5p - 2)$

**4** Expand $\frac{1}{2}(2y - 8)$

**5** Expand and simplify.

$5p(p^2 + 6p) - 9p(2p - 3)$

**6** Expand and simplify.

**a** $(x + 4)(x + 5)$ **b** $(x + 7)(x - 2)$

c $(5x - 3)(2x - 5)$ **d** $(3x + 4y)(5y + 6x)$

**e** $(2x - 7)^2$

**7** Expand and simplify.

**a** $(x + 4)(x + 5)(x + 6)$ **b** $(x + 7)(x - 2)(x + 3)$

**c** $(2x + 3)(x - 1)(3x + 4)$ **d** $(3x - 2)^2(7 + 4x)$

e $(2x - 3)^3$

**Extend**

**8** Expand and simplify $(x + 3)^2 + (x - 4)^2$

**9** Expand and simplify.

**a** $\left(x+\frac{1}{x}\right)\left(x-\frac{2}{x}\right)$ **b** $\left(x+\frac{1}{x}\right)^2$

# Surds and rationalising the denominator

| **A LEVEL LINKS** | |
|---|---|
| Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds | |
| **Video Links** | |
| **Corbett Maths** | **Hegarty Maths on Youtube** |
| 305 Surds: Intro, rules, simplifying<br>306 Surds: Addition/subtraction<br>307 Surds: Rationalising the denominator<br>308 Surds: Expanding brackets | Simplifying surds<br>Brackets involving surds 1<br>Brackets involving surds 2<br>Rationalising surds 1<br>Rationalising surds 2 |

## Key points

- A surd is the square root of a number that is not a square number, for example $\sqrt{2}, \sqrt{3}, \sqrt{5},$ etc.
- Surds can be used to give the exact value for an answer.
- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
- To rationalise the denominator means to remove the surd from the denominator of a fraction.
- To rationalise $\frac{a}{\sqrt{b}}$ you multiply the numerator and denominator by the surd $\sqrt{b}$
- To rationalise $\frac{a}{b+\sqrt{c}}$ you multiply the numerator and denominator by $b-\sqrt{c}$

## Examples

**Example 1** Simplify $\sqrt{50}$

| | |
|---|---|
| $\sqrt{50} = \sqrt{25 \times 2}$ | **1** Choose two numbers that are factors of 50. One of the factors must be a square number |
| $= \sqrt{25} \times \sqrt{2}$ | **2** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ |
| $= 5 \times \sqrt{2}$ | **3** Use $\sqrt{25} = 5$ |
| $= 5\sqrt{2}$ | |

**Example 2** Simplify $\sqrt{147} - 2\sqrt{12}$

| | |
|---|---|
| $\sqrt{147} - 2\sqrt{12}$<br>$= \sqrt{49 \times 3} - 2\sqrt{4 \times 3}$ | **1** Simplify $\sqrt{147}$ and $2\sqrt{12}$. Choose two numbers that are factors of 147 and two numbers that are factors of 12. One of each pair of factors must be a square number |
| $= \sqrt{49} \times \sqrt{3} - 2\sqrt{4} \times \sqrt{3}$ | **2** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ |
| $= 7 \times \sqrt{3} - 2 \times 2 \times \sqrt{3}$ | **3** Use $\sqrt{49} = 7$ and $\sqrt{4} = 2$ |
| $= 7\sqrt{3} - 4\sqrt{3}$<br>$= 3\sqrt{3}$ | **4** Collect like terms |

**Example 3** Simplify $\left(\sqrt{7}+\sqrt{2}\right)\left(\sqrt{7}-\sqrt{2}\right)$

| | |
|---|---|
| $\left(\sqrt{7}+\sqrt{2}\right)\left(\sqrt{7}-\sqrt{2}\right)$<br>$= \sqrt{49}-\sqrt{7}\sqrt{2}+\sqrt{2}\sqrt{7}-\sqrt{4}$<br><br>$= 7 - 2$<br>$= 5$ | **1** Expand the brackets. A common mistake here is to write $\left(\sqrt{7}\right)^2 = 49$<br><br>**2** Collect like terms:<br>$-\sqrt{7}\sqrt{2}+\sqrt{2}\sqrt{7}$<br>$= -\sqrt{7}\sqrt{2}+\sqrt{7}\sqrt{2} = 0$ |

**Example 4** Rationalise $\frac{1}{\sqrt{3}}$

| | |
|---|---|
| $\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$<br><br>$= \frac{1 \times \sqrt{3}}{\sqrt{9}}$<br><br>$= \frac{\sqrt{3}}{3}$ | **1** Multiply the numerator and denominator by $\sqrt{3}$<br><br>**2** Use $\sqrt{9} = 3$ |

**Example 5** Rationalise and simplify $\frac{\sqrt{2}}{\sqrt{12}}$

| | |
|---|---|
| $\frac{\sqrt{2}}{\sqrt{12}} = \frac{\sqrt{2}}{\sqrt{12}} \times \frac{\sqrt{12}}{\sqrt{12}}$<br><br>$= \frac{\sqrt{2} \times \sqrt{4 \times 3}}{12}$<br><br>$= \frac{2\sqrt{2}\sqrt{3}}{12}$<br><br>$= \frac{\sqrt{2}\sqrt{3}}{6}$ | **1** Multiply the numerator and denominator by $\sqrt{12}$<br><br>**2** Simplify $\sqrt{12}$ in the numerator. Choose two numbers that are factors of 12. One of the factors must be a square number<br><br>**3** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$<br>**4** Use $\sqrt{4} = 2$<br><br>**5** Simplify the fraction: $\frac{2}{12}$ simplifies to $\frac{1}{6}$ |

**Example 6** Rationalise and simplify $\frac{3}{2+\sqrt{5}}$

| | |
|---|---|
| $\frac{3}{2+\sqrt{5}} = \frac{3}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$ | **1** Multiply the numerator and denominator by $2-\sqrt{5}$ |
| $= \frac{3\left(2-\sqrt{5}\right)}{\left(2+\sqrt{5}\right)\left(2-\sqrt{5}\right)}$ | **2** Expand the brackets |
| $= \frac{6-3\sqrt{5}}{4+2\sqrt{5}-2\sqrt{5}-5}$ | **3** Simplify the fraction |
| $= \frac{6-3\sqrt{5}}{-1}$ | **4** Divide the numerator by −1 Remember to change the sign of all terms when dividing by −1 |
| $= 3\sqrt{5}-6$ | |

# Practice – NON-CALCULATOR

**For each question, show each step of your working.**

**Hint**
One of the two numbers you choose at the start must be a square number.

**1** Simplify.

**a** $\sqrt{125}$ **b** $\sqrt{48}$

c $\sqrt{162}$

**Watch out!**
Check you have chosen the highest square number at the start.

**2** Simplify.

**a** $\sqrt{72}+\sqrt{162}$ **b** $\sqrt{50}-\sqrt{8}$

c $2\sqrt{12}-\sqrt{12}+\sqrt{27}$

**3** Expand and simplify.

**a** $(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})$ b $(5+\sqrt{2})(6-\sqrt{8})$

**4** Rationalise and simplify, if possible.

**a** $\frac{1}{\sqrt{5}}$ b $\frac{2}{\sqrt{7}}$

**c** $\frac{\sqrt{8}}{\sqrt{24}}$ **d** $\frac{\sqrt{5}}{\sqrt{45}}$

**5** Rationalise and simplify.

**a** $\frac{1}{3-\sqrt{5}}$ **b** $\frac{2}{4+\sqrt{3}}$

# Rules of indices

| **A LEVEL LINKS** | |
|---|---|
| Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds | |
| **Video Links** | |
| **Corbett Maths** | **Hegarty Maths on Youtube** |
| 173 Indices: Fractional<br>174 Indices: Laws of<br>175 Indices: Negative | Index form 3 (power of negative integers)<br>Index form 7 (power of unit fractions)<br>Index form 8 (power of non-unit fractions)<br>Index form 9 (combination of rules) |

## Key points

- $a^m \times a^n = a^{m+n}$
- $\dfrac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$
- $a^{\frac{1}{n}} = \sqrt[n]{a}$ i.e. the *n*th root of *a*
- $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$
- $a^{-m} = \dfrac{1}{a^m}$
- The square root of a number produces two solutions, e.g. $\sqrt{16} = \pm 4$.

## Examples

**Example 1** Evaluate $10^0$

| | |
|---|---|
| $10^0 = 1$ | Any value raised to the power of zero is equal to 1 |

**Example 2** Evaluate $9^{\frac{1}{2}}$

| | |
|---|---|
| $9^{\frac{1}{2}} = \sqrt{9}$<br>$= 3$ | Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$ |

**Example 3** Evaluate $27^{\frac{2}{3}}$

| | |
|---|---|
| $27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2$<br>$= 3^2$<br>$= 9$ | **1** Use the rule $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$<br>**2** Use $\sqrt[3]{27} = 3$ |

**Example 4** Evaluate $4^{-2}$

| | |
|---|---|
| $4^{-2} = \frac{1}{4^2}$ <br> $= \frac{1}{16}$ | **1** Use the rule $a^{-m} = \frac{1}{a^m}$ <br> **2** Use $4^2 = 16$ |

**Example 5** Simplify $\frac{6x^5}{2x^2}$

| | |
|---|---|
| $\frac{6x^5}{2x^2} = 3x^3$ | $6 \div 2 = 3$ and use the rule $\frac{a^m}{a^n} = a^{m-n}$ to give $\frac{x^5}{x^2} = x^{5-2} = x^3$ |

**Example 6** Simplify $\frac{x^3 \times x^5}{x^4}$

| | |
|---|---|
| $\frac{x^3 \times x^5}{x^4} = \frac{x^{3+5}}{x^4} = \frac{x^8}{x^4}$ <br> $= x^{8-4} = x^4$ | **1** Use the rule $a^m \times a^n = a^{m+n}$ <br> **2** Use the rule $\frac{a^m}{a^n} = a^{m-n}$ |

**Example 7** Write $\frac{1}{3x}$ as a single power of $x$

| | |
|---|---|
| $\frac{1}{3x} = \frac{1}{3}x^{-1}$ | Use the rule $\frac{1}{a^m} = a^{-m}$, note that the fraction $\frac{1}{3}$ remains unchanged |

**Example 8** Write $\frac{4}{\sqrt{x}}$ as a single power of $x$

| | |
|---|---|
| $\frac{4}{\sqrt{x}} = \frac{4}{x^{\frac{1}{2}}}$ <br> $= 4x^{-\frac{1}{2}}$ | **1** Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$ <br> **2** Use the rule $\frac{1}{a^m} = a^{-m}$ |

## Practice

1 Evaluate.

**a** $14^0$ **b** $49^{\frac{1}{2}}$ **c** $64^{\frac{1}{3}}$ **d** $25^{\frac{3}{2}}$

**e** $8^{\frac{5}{3}}$ **f** $5^{-2}$ **g** $4^{-3}$

2 Simplify.

**a** $\dfrac{3x \times 2x^3}{2x^3}$ **b** $\dfrac{7x^3y^2}{14x^5y}$

**c** $\dfrac{c^{\frac{1}{2}}}{c^2 \times c^{\frac{3}{2}}}$ **d** $\dfrac{\left(2x^2\right)^3}{4x^0}$

**e** $\dfrac{x^{\frac{1}{2}} \times x^{\frac{3}{2}}}{x^{-2} \times x^3}$

> **Watch out!**
> Remember that any value raised to the power of zero is 1. This is the rule $a^0 = 1$.

**3** Evaluate.

**a** $27^{-\frac{2}{3}}$ **b** $9^{-\frac{1}{2}} \times 2^3$ **c** $\left(\dfrac{9}{16}\right)^{-\frac{1}{2}}$ **d** $\left(\dfrac{27}{64}\right)^{-\frac{2}{3}}$

**4** Write the following as a single power of $x$.

**a** $\dfrac{1}{x^7}$ **b** $\sqrt[4]{x}$ **c** $\sqrt[5]{x^2}$ **d** $\dfrac{1}{\sqrt[3]{x^2}}$

**5** Write the following without negative or fractional powers.

**a** $x^{-3}$ **b** $x^{\frac{2}{5}}$ **c** $x^{-\frac{3}{4}}$

**6** Write the following in the form $ax^n$.

**a** $5\sqrt{x}$ **b** $\dfrac{2}{x^3}$ **c** $\dfrac{1}{3x^4}$ d $\dfrac{4}{\sqrt[3]{x}}$

**7** Write as sums of powers of $x$.

**a** $\dfrac{x^5+1}{x^2}$ **b** $x^2\left(x+\dfrac{1}{x}\right)$ **c** $x^{-4}\left(x^2+\dfrac{1}{x^3}\right)$

# Factorising expressions

| A LEVEL LINKS | |
|---|---|
| Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants | |
| **Video Links** | |
| **Corbett Maths** | **Hegarty Maths on Youtube** |
| 118 Factorisation: quadratics<br>119 Factorisation: quadratics harder<br>120 Factorisation: Difference of two squares<br>24 Algebraic fractions: simplifying | Factorise quadratic expressions 5<br>Factorise quadratic expressions 6<br>Simplify algebraic fractions (involving quadratics) |

## Key points

- Factorising an expression is the opposite of expanding the brackets.
- A quadratic expression is in the form $ax^2 + bx + c$, where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is $b$ and whose product is $ac$.
- An expression in the form $x^2 – y^2$ is called the difference of two squares. It factorises to $(x – y)(x + y)$.

## Examples

**Example 1** Factorise $15x^2y^3 + 9x^4y$

| | |
|---|---|
| $15x^2y^3 + 9x^4y = 3x^2y(5y^2 + 3x^2)$ | The highest common factor is $3x^2y$. So take $3x^2y$ outside the brackets and then divide each term by $3x^2y$ to find the terms in the brackets |

**Example 2** Factorise $4x^2 – 25y^2$

| | |
|---|---|
| $4x^2 – 25y^2 = (2x + 5y)(2x − 5y)$ | This is the difference of two squares as the two terms can be written as $(2x)^2$ and $(5y)^2$ |

**Example 3** Factorise $x^2 + 3x – 10$

| | |
|---|---|
| $b = 3$, $ac = −10$ | **1** Work out the two factors of $ac = −10$ which add to give $b = 3$ (5 and −2) |
| So $x^2 + 3x – 10 = x^2 + 5x – 2x – 10$ | **2** Rewrite the $b$ term ($3x$) using these two factors |
| $= x(x + 5) – 2(x + 5)$ | **3** Factorise the first two terms and the last two terms |
| $= (x + 5)(x – 2)$ | **4** $(x + 5)$ is a factor of both terms |

**Example 4** Factorise $6x^2 - 11x - 10$

| | |
|---|---|
| $b = -11$, $ac = -60$<br><br>So<br>$6x^2 - 11x - 10 = 6x^2 - 15x + 4x - 10$<br><br>$= 3x(2x - 5) + 2(2x - 5)$<br><br>$= (2x - 5)(3x + 2)$ | **1** Work out the two factors of $ac = -60$ which add to give $b = -11$ ($-15$ and 4)<br>**2** Rewrite the $b$ term ($-11x$) using these two factors<br>**3** Factorise the first two terms and the last two terms<br>**4** $(2x - 5)$ is a factor of both terms |

**Example 5** Simplify $\frac{x^2 - 4x - 21}{2x^2 + 9x + 9}$

| | |
|---|---|
| $\frac{x^2 - 4x - 21}{2x^2 + 9x + 9}$ | **1** Factorise the numerator and the denominator |
| For the numerator:<br>$b = -4$, $ac = -21$<br><br>So<br>$x^2 - 4x - 21 = x^2 - 7x + 3x - 21$<br><br>$= x(x - 7) + 3(x - 7)$<br><br>$= (x - 7)(x + 3)$ | **2** Work out the two factors of $ac = -21$ which add to give $b = -4$ ($-7$ and 3)<br><br>**3** Rewrite the $b$ term ($-4x$) using these two factors<br>**4** Factorise the first two terms and the last two terms<br>**5** $(x - 7)$ is a factor of both terms |
| For the denominator:<br>$b = 9$, $ac = 18$<br><br>So<br>$2x^2 + 9x + 9 = 2x^2 + 6x + 3x + 9$<br><br>$= 2x(x + 3) + 3(x + 3)$<br><br>$= (x + 3)(2x + 3)$ | **6** Work out the two factors of $ac = 18$ which add to give $b = 9$ (6 and 3)<br><br>**7** Rewrite the $b$ term ($9x$) using these two factors<br>**8** Factorise the first two terms and the last two terms<br>**9** $(x + 3)$ is a factor of both terms |
| So<br>$\frac{x^2 - 4x - 21}{2x^2 + 9x + 9} = \frac{(x-7)(x+3)}{(x+3)(2x+3)}$<br>$= \frac{x-7}{2x+3}$ | **10** $(x + 3)$ is a factor of both the numerator and denominator so cancels out as a value divided by itself is 1 |

# Practice

**Hint**
Take the highest common factor outside the bracket.

**1** Factorise.

**a** $6x^4y^3 - 10x^3y^4$ **b** $21a^3b^5 + 35a^5b^2$

**2** Factorise

**a** $x^2 + 7x + 12$ **b** $x^2 - 11x + 30$

**c** $x^2 - 7x - 18$ **d** $x^2 - 3x - 40$

**3** Factorise

**a** $36x^2 - 49y^2$ **b** $18a^2 - 200b^2c^2$

**4** Factorise

**a** $2x^2 + x - 3$ **b** $6x^2 + 17x + 5$

**c** $9x^2 - 15x + 4$ d $12x^2 - 38x + 20$

**5** Simplify the algebraic fractions.

**a** $\dfrac{2x^2+4x}{x^2-x}$ b $\dfrac{x^2-2x-8}{x^2-4x}$ **c** $\dfrac{x^2-5x}{x^2-25}$

**d** $\dfrac{2x^2+14x}{2x^2+4x-70}$

**6** Simplify

**a** $\dfrac{9x^2-16}{3x^2+17x-28}$ **b** $\dfrac{2x^2-7x-15}{3x^2-17x+10}$

**Extend**

**7** Simplify $\sqrt{x^2+10x+25}$

**8.** Simplify $\dfrac{(x+2)^2+3(x+2)^2}{x^2-4}$

# Completing the square

| **A LEVEL LINKS** | |
|---|---|
| Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants | |
| **Video Links** | |
| **Corbett Maths** | **Hegarty Maths on Youtube** |
| 10 Algebra: Completing the square | Completing the square 1<br>Completing the square 2<br>Completing the square 3 |

## Key points

- Completing the square for a quadratic rearranges $ax^2 + bx + c$ into the form $p(x + q)^2 + r$
- If $a \neq 1$, then factorise using $a$ as a common factor.
-

## Examples

**Example 1** Complete the square for the quadratic expression $x^2 + 6x - 2$

| | |
|---|---|
| $x^2 + 6x - 2$<br><br>$= (x + 3)^2 - 9 - 2$<br><br>$= (x + 3)^2 - 11$ | **1** Write $x^2 + bx + c$ in the form $\left(x+\frac{b}{2}\right)^2-\left(\frac{b}{2}\right)^2+c$<br>**2** Simplify |

**Example 2** Write $2x^2 - 5x + 1$ in the form $p(x + q)^2 + r$

| | |
|---|---|
| $2x^2 - 5x + 1$<br><br>$= 2\left(x^2-\frac{5}{2}x\right)+1$<br><br>$= 2\left[\left(x-\frac{5}{4}\right)^2-\left(\frac{5}{4}\right)^2\right]+1$<br><br>$= 2\left(x-\frac{5}{4}\right)^2-\frac{25}{8}+1$<br><br>$= 2\left(x-\frac{5}{4}\right)^2-\frac{17}{8}$ | **1** Before completing the square write $ax^2 + bx + c$ in the form $a\left(x^2+\frac{b}{a}x\right)+c$<br>**2** Now complete the square by writing $x^2-\frac{5}{2}x$ in the form $\left(x+\frac{b}{2}\right)^2-\left(\frac{b}{2}\right)^2$<br>**3** Expand the square brackets – don't forget to multiply $\left(\frac{5}{4}\right)^2$ by the factor of 2<br>**4** Simplify |

## Practice

**1** Write the following quadratic expressions in the form $(x + p)^2 + q$

**a** $x^2 + 4x + 3$ **b** $x^2 – 8x$ **c** $x^2 – 2x + 7$

**2** Write the following quadratic expressions in the form $p(x + q)^2 + r$

**a** $2x^2 – 8x – 16$ b $3x^2 + 12x – 9$

**3** Complete the square.

**a** $2x^2 + 3x + 6$ **b** $3x^2 + 5x + 3$

## Extend

**4** Write $(25x^2 + 30x + 12)$ in the form $(ax + b)^2 + c$.

# Solving quadratic equations by factorisation

| A LEVEL LINKS | |
|---|---|
| Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants | |
| **Video Links** | |
| **Corbett Maths** | **Hegarty Maths on Youtube** |
| 266 Quadratics: solving(factorising) | Solving quadratic equations 3 (by factorising)<br>Solving quadratic equations 4 (by factorising) |

## Key points

- A quadratic equation is an equation in the form $ax^2 + bx + c = 0$ where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is $b$ and whose products is $ac$.
- When the product of two numbers is 0, then at least one of the numbers must be 0.
- If a quadratic can be solved it will have two solutions (these may be equal).

## Examples

**Example 1** Solve $5x^2 = 15x$

| | |
|---|---|
| $5x^2 = 15x$<br><br>$5x^2 - 15x = 0$ | **1** Rearrange the equation so that all of the terms are on one side of the equation and it is equal to zero. Do not divide both sides by $x$ as this would lose the solution $x = 0$. |
| $5x(x - 3) = 0$ | **2** Factorise the quadratic equation. $5x$ is a common factor. |
| So $5x = 0$ or $(x - 3) = 0$ | **3** When two values multiply to make zero, at least one of the values must be zero. |
| Therefore $x = 0$ or $x = 3$ | **4** Solve these two equations. |

**Example 2** Solve $x^2 + 7x + 12 = 0$

| | |
|---|---|
| $x^2 + 7x + 12 = 0$<br><br>$b = 7, ac = 12$ | **1** Factorise the quadratic equation. Work out the two factors of $ac = 12$ which add to give you $b = 7$. (4 and 3) |
| $x^2 + 4x + 3x + 12 = 0$ | **2** Rewrite the $b$ term ($7x$) using these two factors. |
| $x(x + 4) + 3(x + 4) = 0$ | **3** Factorise the first two terms and the last two terms. |
| $(x + 4)(x + 3) = 0$ | **4** $(x + 4)$ is a factor of both terms. |
| So $(x + 4) = 0$ or $(x + 3) = 0$ | **5** When two values multiply to make zero, at least one of the values must be zero. |
| Therefore $x = -4$ or $x = -3$ | **6** Solve these two equations. |

**Example 3** Solve $9x^2 - 16 = 0$

| | |
|---|---|
| $9x^2 - 16 = 0$<br>$(3x + 4)(3x - 4) = 0$ | **1** Factorise the quadratic equation. This is the difference of two squares as the two terms are $(3x)^2$ and $(4)^2$. |
| So $(3x + 4) = 0$ or $(3x - 4) = 0$ | **2** When two values multiply to make zero, at least one of the values must be zero. |
| $x = -\frac{4}{3}$ or $x = \frac{4}{3}$ | **3** Solve these two equations. |

**Example 4** Solve $2x^2 - 5x - 12 = 0$

| | |
|---|---|
| $b = -5$, $ac = -24$ | **1** Factorise the quadratic equation. Work out the two factors of $ac = -24$ which add to give you $b = -5$. ($-8$ and 3) |
| So $2x^2 - 8x + 3x - 12 = 0$ | **2** Rewrite the $b$ term ($-5x$) using these two factors. |
| $2x(x - 4) + 3(x - 4) = 0$ | **3** Factorise the first two terms and the last two terms. |
| $(x - 4)(2x + 3) = 0$ | **4** $(x - 4)$ is a factor of both terms. |
| So $(x - 4) = 0$ or $(2x + 3) = 0$ | **5** When two values multiply to make zero, at least one of the values must be zero. |
| $x = 4$ or $x = -\frac{3}{2}$ | **6** Solve these two equations. |

# Practice

**1** Solve

**a** $6x^2 + 4x = 0$ b $x^2 + 7x + 10 = 0$ **c** $x^2 - 10x + 24 = 0$

**d** $x^2 - 36 = 0$ **e** $x^2 + 3x - 28 = 0$ **f** $2x^2 - 7x - 4 = 0$

**2** Solve

**a** $x^2 - 3x = 10$ **b** $x^2 + 5x = 24$

**c** $x(x + 2) = 2x + 25$ **d** $x(3x + 1) = x^2 + 15$

**Hint**
Get all terms onto one side of the equation.

# Solving quadratic equations by completing the square

| **A LEVEL LINKS** | |
|---|---|
| Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants | |
| **Video Links** | |
| **Corbett Maths** | **Hegarty Maths on Youtube** |
| 267a Quadratics: solving (completing the square) | Solving by completing the square 1<br>Solving by completing the square 2 |

## Key points

- Completing the square lets you write a quadratic equation in the form $p(x + q)^2 + r = 0$.

## Examples

**Example 5** Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

| | |
|---|---|
| $x^2 + 6x + 4 = 0$ | **1** Write $x^2 + bx + c = 0$ in the form $\left(x+\frac{b}{2}\right)^2-\left(\frac{b}{2}\right)^2+c=0$ |
| $(x + 3)^2 - 9 + 4 = 0$ | |
| $(x + 3)^2 - 5 = 0$ | **2** Simplify. |
| $(x + 3)^2 = 5$ | **3** Rearrange the equation to work out $x$. First, add 5 to both sides. |
| $x + 3 = \pm\sqrt{5}$ | **4** Square root both sides. Remember that the square root of a value gives two answers. |
| $x = \pm\sqrt{5} - 3$ | **5** Subtract 3 from both sides to solve the equation. |
| So $x = -\sqrt{5} - 3$ or $x = \sqrt{5} - 3$ | **6** Write down both solutions. |

**Example 6** Solve $2x^2 - 7x + 4 = 0$. Give your solutions in surd form.

| | |
|---|---|
| $2x^2 - 7x + 4 = 0$ | **1** Before completing the square write $ax^2 + bx + c$ in the form $a\left(x^2+\frac{b}{a}x\right)+c$ |
| $2\left(x^2-\frac{7}{2}x\right)+4=0$ | |
| $2\left[\left(x-\frac{7}{4}\right)^2-\left(\frac{7}{4}\right)^2\right]+4=0$ | **2** Now complete the square by writing $x^2-\frac{7}{2}x$ in the form $\left(x+\frac{b}{2a}\right)^2-\left(\frac{b}{2a}\right)^2$ |
| $2\left(x-\frac{7}{4}\right)^2-\frac{49}{8}+4=0$ | **3** Expand the square brackets. |
| $2\left(x-\frac{7}{4}\right)^2-\frac{17}{8}=0$ | **4** Simplify. |
| | *(continued on next page)* |
| $2\left(x-\frac{7}{4}\right)^2=\frac{17}{8}$ | |

| | |
|---|---|
| $\left(x-\frac{7}{4}\right)^2=\frac{17}{16}$ | **5** Rearrange the equation to work out $x$. First, add $\frac{17}{8}$ to both sides. |
| $x-\frac{7}{4}=\pm\frac{\sqrt{17}}{4}$ | **6** Divide both sides by 2. |
| $x=\pm\frac{\sqrt{17}}{4}+\frac{7}{4}$ | **7** Square root both sides. Remember that the square root of a value gives two answers. |
| So $x=\frac{7}{4}-\frac{\sqrt{17}}{4}$ or $x=\frac{7}{4}+\frac{\sqrt{17}}{4}$ | **8** Add $\frac{7}{4}$ to both sides. |
| | **9** Write down both the solutions. |

# Practice

**1** Solve by completing the square.

**a** $x^2 - 4x - 3 = 0$ **b** $x^2 + 8x - 5 = 0$ **c** $2x^2 + 8x - 5 = 0$

**2** Solve by completing the square.

$(x - 4)(x + 2) = 5$

**Hint**
Get all terms onto one side of the equation.

# Solving quadratic equations by using the formula

| **A LEVEL LINKS** | |
|---|---|
| Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants | |
| **Video Links** | |
| **Corbett Maths** | **Hegarty Maths on Youtube** |
| 267 Quadratics: formula | Solving using the quadratic formula 1<br>Solving using the quadratic formula 2 |

## Key points

- Any quadratic equation of the form $ax^2 + bx + c = 0$ can be solved using the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- If $b^2 - 4ac$ is negative then the quadratic equation does not have any real solutions.
- It is useful to write down the formula before substituting the values for $a$, $b$ and $c$.

## Examples

**Example 7** Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

| | |
|---|---|
| $a = 1, b = 6, c = 4$<br>$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ | **1** Identify $a$, $b$ and $c$ and write down the formula.<br>Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it. |
| $x = \frac{-6 \pm \sqrt{6^2 - 4(1)(4)}}{2(1)}$ | **2** Substitute $a = 1$, $b = 6$, $c = 4$ into the formula. |
| $x = \frac{-6 \pm \sqrt{20}}{2}$ | **3** Simplify. The denominator is 2, but this is only because $a = 1$. The denominator will not always be 2. |
| $x = \frac{-6 \pm 2\sqrt{5}}{2}$ | **4** Simplify $\sqrt{20}$.<br>$\sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$ |
| $x = -3 \pm \sqrt{5}$ | **5** Simplify by dividing numerator and denominator by 2. |
| So $x = -3 - \sqrt{5}$ or $x = \sqrt{5} - 3$ | **6** Write down both the solutions. |

**Example 8** Solve $3x^2 - 7x - 2 = 0$. Give your solutions in surd form.

| | |
|---|---|
| $a = 3$, $b = -7$, $c = -2$<br>$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ | **1** Identify $a$, $b$ and $c$, making sure you get the signs right and write down the formula.<br>Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it. |
| $x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-2)}}{2(3)}$ | **2** Substitute $a = 3$, $b = -7$, $c = -2$ into the formula. |
| $x = \frac{7 \pm \sqrt{73}}{6}$ | **3** Simplify. The denominator is 6 when $a = 3$. A common mistake is to always write a denominator of 2. |
| So $x = \frac{7 - \sqrt{73}}{6}$ or $x = \frac{7 + \sqrt{73}}{6}$ | **4** Write down both the solutions. |

# Practice

**1** Solve, giving your solutions in surd form.

**a** $3x^2 + 6x + 2 = 0$ **b** $2x^2 - 4x - 7 = 0$

**2** Solve the equation $x^2 - 7x + 2 = 0$

Give your solutions in the form $\frac{a \pm \sqrt{b}}{c}$, where $a$, $b$ and $c$ are integers.

**3** Solve $10x^2 + 3x + 3 = 5$

Give your solution in surd form.

**Hint**
Get all terms onto one side of the equation.

# Solving linear simultaneous equations using the elimination method

| **A LEVEL LINKS** | |
|---|---|
| Scheme of work: 1c. Equations – quadratic/linear simultaneous | |
| **Video Links** | |
| **Corbett Maths** | **Hegarty Maths on Youtube** |
| 295 Simultaneous equations (elimination) | Simultaneous equations by elimination 4 |

## Key points

- Two equations are simultaneous when they are both true at the same time.
- Solving simultaneous linear equations in two unknowns involves finding the value of each unknown which works for both equations.
- Make sure that the coefficient of one of the unknowns is the same in both equations.
- Eliminate this equal unknown by either subtracting or adding the two equations.

## Examples

**Example 1** Solve the simultaneous equations $3x + y = 5$ and $x + y = 1$

| | |
|---|---|
| $3x + y = 5$<br>$-\quad x + y = 1$<br>$2x \quad = 4$<br>So $x = 2$ | **1** Subtract the second equation from the first equation to eliminate the $y$ term. |
| Using $x + y = 1$<br>$2 + y = 1$<br>So $y = -1$ | **2** To find the value of $y$, substitute $x = 2$ into one of the original equations. |
| Check:<br>equation 1: $3 \times 2 + (-1) = 5$ YES<br>equation 2: $2 + (-1) = 1$ YES | **3** Substitute the values of $x$ and $y$ into both equations to check your answers. |

**Example 2** Solve $x + 2y = 13$ and $5x - 2y = 5$ simultaneously.

| | |
|---|---|
| $x + 2y = 13$<br>$+\quad 5x - 2y = 5$<br>$6x \quad = 18$<br>So $x = 3$ | **1** Add the two equations together to eliminate the $y$ term. |
| Using $x + 2y = 13$<br>$3 + 2y = 13$<br>So $y = 5$ | **2** To find the value of $y$, substitute $x = 3$ into one of the original equations. |
| Check:<br>equation 1: $3 + 2 \times 5 = 13$ YES<br>equation 2: $5 \times 3 - 2 \times 5 = 5$ YES | **3** Substitute the values of $x$ and $y$ into both equations to check your answers. |

**Example 3** Solve $2x + 3y = 2$ and $5x + 4y = 12$ simultaneously.

| | |
|---|---|
| $(2x + 3y = 2) \times 4 \rightarrow \quad 8x + 12y = 8$<br>$(5x + 4y = 12) \times 3 \rightarrow \quad 15x + 12y = 36$<br>$7x \quad = 28$<br><br>So $x = 4$ | **1** Multiply the first equation by 4 and the second equation by 3 to make the coefficient of $y$ the same for both equations. Then subtract the first equation from the second equation to eliminate the $y$ term. |
| Using $2x + 3y = 2$<br>$2 \times 4 + 3y = 2$<br>So $y = -2$ | **2** To find the value of $y$, substitute $x = 4$ into one of the original equations. |
| Check:<br>equation 1: $2 \times 4 + 3 \times (-2) = 2$ YES<br>equation 2: $5 \times 4 + 4 \times (-2) = 12$ YES | **3** Substitute the values of $x$ and $y$ into both equations to check your answers. |

# Practice

Solve these simultaneous equations.

**1** $3x + y = 7$
$3x + 2y = 5$

**2** $3x + 4y = 7$
$x - 4y = 5$

**3** $2x + y = 11$
$x - 3y = 9$

**4** $2x + 3y = 11$
$3x + 2y = 4$

# Solving linear simultaneous equations using the substitution method

| | |
|---|---|
| **A LEVEL LINKS**<br>Scheme of work: 1c. Equations – quadratic/linear simultaneous | |
| **Video Links** | |
| **Corbett Maths** | **Hegarty Maths on Youtube** |
| 296 Simultaneous equations (substitution, both linear) | Simultaneous equations by substitution |

## Key points

- The subsitution method is the method most commonly used for A level. This is because it is the method used to solve linear and quadratic simultaneous equations.

## Examples

**Example 4** Solve the simultaneous equations $y = 2x + 1$ and $5x + 3y = 14$

| | |
|---|---|
| $5x + 3(2x + 1) = 14$ | **1** Substitute $2x + 1$ for $y$ into the second equation. |
| $5x + 6x + 3 = 14$<br>$11x + 3 = 14$ | **2** Expand the brackets and simplify. |
| $11x = 11$<br>So $x = 1$ | **3** Work out the value of $x$. |
| Using $y = 2x + 1$<br>$y = 2 \times 1 + 1$<br>So $y = 3$ | **4** To find the value of $y$, substitute $x = 1$ into one of the original equations. |
| Check:<br>equation 1: $3 = 2 \times 1 + 1$ YES<br>equation 2: $5 \times 1 + 3 \times 3 = 14$ YES | **5** Substitute the values of $x$ and $y$ into both equations to check your answers. |

**Example 5** Solve $2x - y = 16$ and $4x + 3y = -3$ simultaneously.

| | |
|---|---|
| $y = 2x - 16$ | **1** Rearrange the first equation. |
| $4x + 3(2x - 16) = -3$ | **2** Substitute $2x - 16$ for $y$ into the second equation. |
| $4x + 6x - 48 = -3$<br>$10x - 48 = -3$ | **3** Expand the brackets and simplify. |
| $10x = 45$<br>So $x = 4\frac{1}{2}$ | **4** Work out the value of $x$. |
| Using $y = 2x - 16$<br>$y = 2 \times 4\frac{1}{2} - 16$<br>So $y = -7$ | **5** To find the value of $y$, substitute $x = 4\frac{1}{2}$ into one of the original equations. |
| Check:<br>equation 1: $2 \times 4\frac{1}{2} - (-7) = 16$ YES<br>equation 2: $4 \times 4\frac{1}{2} + 3 \times (-7) = -3$<br>YES | **6** Substitute the values of $x$ and $y$ into both equations to check your answers. |

## Practice

Solve these simultaneous equations.

**1** $y = x - 4$
$2x + 5y = 43$

**2** $y = 2x - 3$
$5x - 3y = 11$

**3** $3x + 4y = 8$
$2x - y = -13$

**4** $3y = 4x - 7$
$2y = 3x - 4$

## Extend

**5** Solve the simultaneous equations $3x + 5y - 20 = 0$ and $2(x + y) = \frac{3(y - x)}{4}$.

# Solving linear and quadratic simultaneous equations

| **A LEVEL LINKS** | |
|---|---|
| Scheme of work: 1c. Equations – quadratic/linear simultaneous | |
| **Video Links** | |
| **Corbett Maths** | **Hegarty Maths on Youtube** |
| 298 Simultaneous equations (advanced)<br>*12 Algebra: Equation of a circle* | Simultaneous equations involving quadratics<br>*Equation of a circle 1* |

**A LEVEL LINKS**
**Scheme of work:** 1c. Equations – quadratic/linear simultaneous

## Key points

- Make one of the unknowns the subject of the linear equation (rearranging where necessary).
- Use the linear equation to substitute into the quadratic equation.
- There are usually two pairs of solutions.

## Examples

**Example 1** Solve the simultaneous equations $y = x + 1$ and $x^2 + y^2 = 13$

| | |
|---|---|
| $x^2 + (x + 1)^2 = 13$ | **1** Substitute $x + 1$ for $y$ into the second equation. |
| $x^2 + x^2 + x + x + 1 = 13$<br>$2x^2 + 2x + 1 = 13$ | **2** Expand the brackets and simplify. |
| $2x^2 + 2x - 12 = 0$<br>$(2x - 4)(x + 3) = 0$ | **3** Factorise the quadratic equation. |
| So $x = 2$ or $x = -3$ | **4** Work out the values of $x$. |
| Using $y = x + 1$<br>When $x = 2$, $y = 2 + 1 = 3$<br>When $x = -3$, $y = -3 + 1 = -2$ | **5** To find the value of $y$, substitute both values of $x$ into one of the original equations. |
| So the solutions are<br>$x = 2$, $y = 3$ and $x = -3$, $y = -2$ | |
| Check:<br>equation 1: $3 = 2 + 1$ YES<br>and $-2 = -3 + 1$ YES<br>equation 2: $2^2 + 3^2 = 13$ YES<br>and $(-3)^2 + (-2)^2 = 13$ YES | **6** Substitute both pairs of values of $x$ and $y$ into both equations to check your answers. |

**Example 2** Solve $2x + 3y = 5$ and $2y^2 + xy = 12$ simultaneously.

| | |
|---|---|
| $x = \frac{5-3y}{2}$ | **1** Rearrange the first equation. |
| $2y^2 + \left(\frac{5-3y}{2}\right)y = 12$ | **2** Substitute $\frac{5-3y}{2}$ for $x$ into the second equation. Notice how it is easier to substitute for $x$ than for $y$. |
| $2y^2 + \frac{5y-3y^2}{2} = 12$<br>$4y^2 + 5y - 3y^2 = 24$<br>$y^2 + 5y - 24 = 0$ | **3** Expand the brackets and simplify. |
| $(y + 8)(y - 3) = 0$ | **4** Factorise the quadratic equation. |
| So $y = -8$ or $y = 3$ | **5** Work out the values of $y$. |
| Using $2x + 3y = 5$<br>When $y = -8$, $2x + 3 \times (-8) = 5$, $x = 14.5$<br>When $y = 3$, $2x + 3 \times 3 = 5$, $x = -2$<br><br>So the solutions are<br>$x = 14.5$, $y = -8$ and $x = -2$, $y = 3$ | **6** To find the value of $x$, substitute both values of $y$ into one of the original equations. |
| Check:<br>equation 1: $2 \times 14.5 + 3 \times (-8) = 5$ YES<br>and $2 \times (-2) + 3 \times 3 = 5$ YES<br>equation 2: $2\times(-8)^2 + 14.5\times(-8) = 12$ YES<br>and $2 \times (3)^2 + (-2) \times 3 = 12$ YES | **7** Substitute both pairs of values of $x$ and $y$ into both equations to check your answers. |

# Practice

Solve these simultaneous equations.

**1** $y = 2x + 1$
$x^2 + y^2 = 10$

**2** $y = 6 - x$
$x^2 + y^2 = 20$

**5** $y = 3x - 5$
$y = x^2 - 2x + 1$

**6** $y = 2x - 1$
$x^2 + xy = 24$

# Linear inequalities

| A LEVEL LINKS | |
|---|---|
| Scheme of work: 1d. Inequalities – linear and quadratic (including graphical solutions | |
| **Video Links** | |
| **Corbett Maths** | **Hegarty Maths on Youtube** |
| 178 Inequalities: solving (one sign)<br>179 Inequalities: solving (two signs)<br>182 Inequalities: regions | |

## Key points

- Solving linear inequalities uses similar methods to those for solving linear equations.
- When you multiply or divide an inequality by a negative number you need to reverse the inequality sign, e.g. < becomes >.

## Examples

**Example 1** Solve $-8 \le 4x < 16$

| | |
|---|---|
| $-8 \le 4x < 16$<br>$-2 \le x < 4$ | Divide all three terms by 4. |

**Example 2** Solve $4 \le 5x < 10$

| | |
|---|---|
| $4 \le 5x < 10$<br>$\frac{4}{5} \le x < 2$ | Divide all three terms by 5. |

**Example 3** Solve $2x - 5 < 7$

| | |
|---|---|
| $2x - 5 < 7$<br>$2x < 12$<br>$x < 6$ | **1** Add 5 to both sides.<br>**2** Divide both sides by 2. |

**Example 4** Solve $2 - 5x \ge -8$

| | |
|---|---|
| $2 - 5x \ge -8$<br>$-5x \ge -10$<br>$x \le 2$ | **1** Subtract 2 from both sides.<br>**2** Divide both sides by −5. Remember to reverse the inequality when dividing by a negative number. |

**Example 5** Solve $4(x - 2) > 3(9 - x)$

| | |
|---|---|
| $4(x - 2) > 3(9 - x)$<br>$4x - 8 > 27 - 3x$<br>$7x - 8 > 27$<br>$7x > 35$<br>$x > 5$ | **1** Expand the brackets.<br>**2** Add $3x$ to both sides.<br>**3** Add 8 to both sides.<br>**4** Divide both sides by 7. |

# Practice

**1** Solve these inequalities.

**a** $5x - 7 \leq 3$ **b** $5 - 2x < 12$ **c** $\frac{x}{2} \geq 5$ **d** $8 < 3 - \frac{x}{3}$

**2** Solve these inequalities.

**a** $3t + 1 < t + 6$ **b** $2(3n - 1) \geq n + 5$

**3** Solve.

**a** $3(2 - x) > 2(4 - x) + 4$ **b** $5(4 - x) > 3(5 - x) + 2$

# Extend

**4** Find the set of values of $x$ for which $2x + 1 > 11$ and $4x - 2 > 16 - 2x$.

# Algebraic Fractions

| **A LEVEL LINKS** | |
|---|---|
| Scheme of work: | |
| **Video Links** | |
| **Corbett Maths** | **Hegarty Maths on Youtube** |
| 21 Algebraic fractions: addition<br>22 Algebraic fractions: division<br>23 Algebraic fractions:multiplication<br>111 Equations: involving fractions<br>111a Equations: fractional advanced | Solve equations with algebraic fractions |

## Key points

- When adding or subtracting fractions you need a common denominator.
- Factorise where possible to help with simplifying

## Examples

**Example 1**: Simplify:

$$\frac{x+1}{3}-\frac{x-3}{2}$$

- The first thing we need when adding or subtracting is a common denominator:

$$\frac{2(x+1)}{6}-\frac{3(x-3)}{6}$$

- Now express as a single fraction and then simplify the numerator – be careful with signs!!

$$=\frac{2(x+1)-3(x-3)}{6}$$

$$=\frac{2x+2-3x+9}{6}$$

$$=\frac{-x+11}{6}$$

**Example 2**: Solve this equation:

$$\frac{3}{x-1}-\frac{2}{x+1}=1$$

- First write as a single fraction.

$$\frac{3(x+1)-2(x-1)}{(x-1)(x+1)}=1$$

- Then multiply both sides by the denominator to get rid of the fraction.

$$3(x+1)-2(x-1)=(x-1)(x+1)$$

- Expand the brackets and simplify – sometimes you may get a quadratic!!

$$3x + 3 - 2x + 2 = x^2 - 1$$

- As this is a quadratic, rearrange so it is equal to zero.

$$x^2 - x - 6 = 0$$

- Factorise and solve:

$$(x - 3)(x + 2) = 0$$

$$x = 3 \; or - 2$$

# Practice

Simplify:

1. $\frac{x+1}{2} - \frac{x-1}{3}$
2. $\frac{2x-1}{5} + \frac{1-x}{7}$
3. $\frac{6p}{5} - \frac{4p-3q}{3}$

Solve:

4. $\frac{x+1}{2} + \frac{x+2}{5} = 3$
5. $\frac{4x+1}{3} - \frac{x+2}{4} = 2$
6. $\frac{4}{x-2} + \frac{7}{x+1} = 3$

# Rearranging Formulae

| **A LEVEL LINKS** | |
|---|---|
| Scheme of work: 6a. Definition, differentiating polynomials, second derivatives | |
| **Video Links** | |
| **Corbett Maths** | **Hegarty Maths on Youtube** |
| 7 Algebra: changing the subject<br>8 Algebra: changing the subject advanced | Change the subject of the formula 3<br>Change the subject of the formula 4<br>Change the subject of the formula 5<br>Change the subject of the formula 6<br>Change the subject of the formula 7 |

## Key points

- To change the subject of a formula, get the terms containing the subject on one side and everything else on the other side.
- You may need to factorise the terms containing the new subject.

## Examples

**Example 1** Make $t$ the subject of the formula $v = u + at$.

| | |
|---|---|
| $v = u + at$ | **1** Get the terms containing $t$ on one side and everything else on the other side. |
| $v - u = at$<br>$t = \frac{v-u}{a}$ | **2** Divide throughout by $a$. |

**Example 2** Make $t$ the subject of the formula $r = 2t - \pi t$.

| | |
|---|---|
| $r = 2t - \pi t$ | **1** All the terms containing $t$ are already on one side and everything else is on the other side. |
| $r = t(2 - \pi)$ | **2** Factorise as $t$ is a common factor. |
| $t = \frac{r}{2-\pi}$ | **3** Divide throughout by $2 - \pi$. |

**Example 3** Make $t$ the subject of the formula $\frac{t+r}{5} = \frac{3t}{2}$.

| | |
|---|---|
| $\frac{t+r}{5} = \frac{3t}{2}$ | **1** Remove the fractions first by multiplying throughout by 10. |
| $2t + 2r = 15t$<br>$2r = 13t$ | **2** Get the terms containing $t$ on one side and everything else on the other side and simplify. |
| $t = \frac{2r}{13}$ | **3** Divide throughout by 13. |

**Example 4** Make $t$ the subject of the formula $r = \frac{3t+5}{t-1}$.

| | |
|---|---|
| $r = \dfrac{3t+5}{t-1}$ | **1** Remove the fraction first by multiplying throughout by $t - 1$. |
| $r(t - 1) = 3t + 5$ | **2** Expand the brackets. |
| $rt - r = 3t + 5$ | |
| $rt - 3t = 5 + r$ | **3** Get the terms containing $t$ on one side and everything else on the other side. |
| $t(r - 3) = 5 + r$ | **4** Factorise the LHS as $t$ is a common factor. |
| $t = \dfrac{5+r}{r-3}$ | **5** Divide throughout by $r - 3$. |

## Practice

Change the subject of each formula to the letter given in the brackets.

**1** $P = 2l + 2w$ [$w$]

**2** $D = \dfrac{S}{T}$ [$T$]

**3** $u = at - \dfrac{1}{2}t$ [$t$]

**4** $V = ax + 4x$ [$x$]

**5** $x = \dfrac{2a-1}{3-a}$ [$a$]

**6** $e(9 + x) = 2e + 1$ [$e$]

**7** $y = \dfrac{2x+3}{4-x}$ [$x$]

**8** Make $r$ the subject of the following formulae.

**a** $P = \pi r + 2r$ **b** $V = \dfrac{2}{3}\pi r^2 h$

**9** Make $x$ the subject of the following formulae.

**a** $\dfrac{xy}{z} = \dfrac{ab}{cd}$

## Extend

**10** Make $x$ the subject of the following equations.

**a** $\dfrac{p}{q}(sx+t) = x-1$ **b** $\dfrac{p}{q}(ax+2y) = \dfrac{3p}{q^2}(x-y)$

# Solutions:

## Expanding brackets and simplifying expressions

## Answers

**1** **a** $6x - 3$ b $-3xy + 2y^2$

**2** **a** $20k^3 - 48k$ b $10h - 12h^3 - 22h^2$

**3** **a** $-y^2 - 4$ **b** $2p - 7p^2$

4 $y - 4$

**5** $5p^3 + 12p^2 + 27p$

**6** **a** $x^2 + 9x + 20$ **b** $x^2 + 5x - 14$
c $10x^2 - 31x + 15$ **d** $18x^2 + 39xy + 20y^2$
e $4x^2 - 28x + 49$

**7** **a** $x^3+15x^2+74x+120$ b $x^3 + 8x^2 + x - 42$

c $6x^3 + 11x^2 - 5x - 12$ d $36x^3 + 15x^2 - 68x + 28$

e $8x^3 - 36x^2 + 54x - 27$

Extend

**8** $2x^2 - 2x + 25$

**9** **a** $x^2 - 1 - \frac{2}{x^2}$ **b** $x^2 + 2 + \frac{1}{x^2}$

# Surds and rationalising the denominator

## Answers

**1** **a** $5\sqrt{5}$ **b** $4\sqrt{3}$ **c** $9\sqrt{2}$

**2** **a** $15\sqrt{2}$ **b** $3\sqrt{2}$ **c** $5\sqrt{3}$

**3** **a** $-1$ b $26 - 4\sqrt{2}$

4 **a** $\frac{\sqrt{5}}{5}$ **b** $\frac{2\sqrt{7}}{7}$ **c** $\frac{\sqrt{3}}{3}$ **d** $\frac{1}{3}$

5 **a** $\frac{3+\sqrt{5}}{4}$ **b** $\frac{2(4-\sqrt{3})}{13}$

# Rules of indices

## Answers

1 **a** 1 **b** 7 **c** 4 d 125

e 32 f $\frac{1}{25}$ **g** $\frac{1}{64}$

2 **a** $3x$ **b** $\frac{y}{2x^2}$ **c** $c^{-3}$

**d** $2x^6$ **e** $x$

3 **a** $\frac{1}{9}$ **b** $\frac{8}{3}$ **c** $\frac{4}{3}$ **d** $\frac{16}{9}$

4 **a** $x^{-7}$ **b** $x^{\frac{1}{4}}$ **c** $x^{\frac{2}{5}}$ d $x^{-\frac{2}{3}}$

5 **a** $\frac{1}{x^3}$ b $\sqrt[5]{x^2}$ **c** $\frac{1}{\sqrt[4]{x^3}}$

6 **a** $5x^{\frac{1}{2}}$ **b** $2x^{-3}$ **c** $\frac{1}{3}x^{-4}$ d $4x^{-\frac{1}{3}}$

7 **a** $x^3 + x^{-2}$ **b** $x^3 + x$ **c** $x^{-2} + x^{-7}$

# Factorising expressions

## Answers

1 **a** $2x^3y^3(3x - 5y)$ **b** $7a^3b^2(3b^3 + 5a^2)$

2 **a** $(x + 3)(x + 4)$ b $(x - 5)(x - 6)$ c $(x - 9)(x + 2)$

**d** $(x - 8)(x + 5)$

**3** **a** $(6x - 7y)(6x + 7y)$ b $2(3a - 10bc)(3a + 10bc)$

**4** **a** (2x+3)(x-1) b $(3x + 1)(2x + 5)$ **c** $(3x - 1)(3x - 4)$
**d** $2(3x - 2)(2x - 5)$

**5** **a** $\frac{2(x+2)}{x-1}$ **b** $\frac{x+2}{x}$ **c** $\frac{x}{x+5}$
**d** $\frac{x}{x-5}$

**6** **a** $\frac{3x+4}{x+7}$ **b** $\frac{2x+3}{3x-2}$

Extend

**7** $(x + 5)$

**8** $\frac{4(x+2)}{x-2}$

# Completing the square

# Answers

**1** **a** $(x + 2)^2 - 1$ **b** $(x - 4)^2 - 16$ **c** $(x - 1)^2 + 6$

**2** **a** $2(x - 2)^2 - 24$ **b** $3(x + 2)^2 - 21$

**3** **a** $2\left(x+\frac{3}{4}\right)^2+\frac{39}{8}$ **b** $3\left(x-\frac{1}{3}\right)^2-\frac{1}{3}$

Extend

**4** $(5x + 3)^2 + 3$

# Solving Quadratic Equations by factorising

# Answers

**1** **a** $x = 0$ or $x = -\frac{2}{3}$ b $x = -5$ or $x = -2$
**c** $x = 4$ or $x = 6$ **d** $x = -6$ or $x = 6$

**e** $x = -7$ or $x = 4$

**f** $x = -\frac{1}{2}$ or $x = 4$

**2** **a** $x = -2$ or $x = 5$

b $x = -8$ or $x = 3$

**c** $x = -5$ or $x = 5$

d $x = -3$ or $x = 2\frac{1}{2}$

**Solving quadratic equations by completing the square**

**1** **a** $x = 2 + \sqrt{7}$ or $x = 2 - \sqrt{7}$

b $x = -4 + \sqrt{21}$ or $x = -4 - \sqrt{21}$

**c** $x = -2 + \sqrt{6.5}$ or $x = -2 - \sqrt{6.5}$

**2** **a** $x = 1 + \sqrt{14}$ or $x = 1 - \sqrt{14}$

**Solving quadratic equations by using the formula**

**1** **a** $x = -1 + \frac{\sqrt{3}}{3}$ or $x = -1 - \frac{\sqrt{3}}{3}$

**b** $x = 1 + \frac{3\sqrt{2}}{2}$ or $x = 1 - \frac{3\sqrt{2}}{2}$

**2** $x = \frac{7+\sqrt{41}}{2}$ or $x = \frac{7-\sqrt{41}}{2}$

**3** $x = \frac{-3+\sqrt{89}}{20}$ or $x = \frac{-3-\sqrt{89}}{20}$

# Solving linear simultaneous equations using the elimination method

# Answers

**1** $x = 3, y = -2$

**2** $x = 3, y = -\frac{1}{2}$

**3** $x = 6, y = -1$

**4** $x = -2, y = 5$

# Solving linear simultaneous equations using the substitution method

# Answers

1 $x = 9, y = 5$

2 $x = –2, y = –7$

3 $x = –4, y = 5$

4 $x = –2, y = –5$

**Extend**

5 $x = -2\frac{1}{2}, y = 5\frac{1}{2}$

# Solving linear and quadratic simultaneous equations

# Answers

1 $x = 1, y = 3$
$x = -\frac{9}{5}, y = -\frac{13}{5}$

2 $x = 2, y = 4$
$x = 4, y = 2$

3 $x = 3, y = 4$
$x = 2, y = 1$

4 $x = -\frac{8}{3}, y = -\frac{19}{3}$
$x = 3, y = 5$

# Linear inequalities

# Answers

1 **a** $x > 4$ **b** $x > -\frac{7}{2}$ **c** $x \geq 10$ **d** $x < –15$

2 **a** $t < \frac{5}{2}$ **b** $n \geq \frac{7}{5}$

3 **a** $x < –6$ **b** $x < \frac{3}{2}$

**Extend**

**4** $x > 5$ (which also satisfies $x > 3$)

# Algebraic Fractions

# Answers

1. $\frac{x+5}{6}$
2. $\frac{9x-2}{35}$
3. $\frac{-2p+15q}{15}$
4. x = 3
5. X = 2
6. X = $\frac{7\pm\sqrt{37}}{3}$

# Rearranging Formulae

# Answers

**1** $w = \frac{P-2l}{2}$

**2** $T = \frac{S}{D}$

**3** $t = \frac{2u}{2a-1}$

**4** $x = \frac{V}{a+4}$

**5** $a = \frac{3x+1}{x+2}$

**6** $e = \frac{1}{x+7}$

**7** $x = \frac{4y-3}{2+y}$

**8** **a** $r = \frac{P}{\pi+2}$

**b** $r = \sqrt{\frac{3V}{2\pi h}}$

**9** **a** $x = \frac{abz}{cdy}$

**Extend**

**17** **a** $x = \frac{q+pt}{q-ps}$

**b** $x = \frac{3py+2pqy}{3p-apq} = \frac{y(3+2q)}{3-aq}$